Maximum principle for the fractional derivatives and its applications to fractional nonlinear problems

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Abstract: Recently, with the development of fractional differential equations, the extremum principles for fractional differential equations have started to draw attention. This motivates us to consider the extremum principle for the Caputo and Hadamard derivatives.

In this paper we obtain new estimates of the Caputo and Hadamard fractional derivatives of a function at its extreme points. The extremum principle is then applied to show that the initial-boundary-value problem for linear and nonlinear time-fractional diffusion equations possesses at most one classical solution and this solution depends continuously on the initial and boundary conditions. The extremum principle for an elliptic equation with a fractional derivative is also proved.

An investigation of the maximum principle for time-fractional diffusion and fractional elliptic equations is devoted to [1-5].

References

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