On the spectral properties of a differential operator arising in the theory of Brownian motion

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Abstract: It is well known that in stochastic processes is widely used the following generalized Ornstein-Uhlenbeck operator

$$Au = -\Delta u + \nabla u \cdot b(x) + c(x)u,$$

where $x \in R^n$. So, A is the generator of the transition semigroup of the stochastic process, which determines the n-dimensional Brownian motion with identity covariant matrix.

The vector-valued function b is called a drift. If $b = 0$, then A is the Schrodinger operator, which has been systematically studied for a long time in connection with applications in quantum mechanics. If $b \neq 0$ and it is unbounded, then the properties of A are significally differ. For example, the well-posedness and spectral properties of A depend on the relationship between the growth at infinity of $|b|$ and the norm of c. For more details, we refer to papers of A. Lunardi and V. Vespri (1997), G. Metafune (2001) and P.J. Rabier (2005).

In this work we discuss the discreteness spectrum criterion of an folloving one-dimentional operator

$$Ly = -y'' + r(x)y' + q(x)y,$$

given in $L_p(R)$ $(1 < p < \infty)$, where r is a continuously differentiable function, and q is a continuous function, and the growth of r not depends on q. To get the desired result, we first give the conditions for invertibility of L and Â describe its domain $D(L)$.

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