# Duhamel principle for the time-fractional diffusion equation in unbounded domain

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**Abstract:** In this paper we establish a fractional Duhamel principle for the time-fractional diffusion equation

$$u_t\left(x,t\right)-\frac{\partial^2}{\partial x^2}D_t^{1-\alpha}u\left(x,t\right)=f(x,t),\ 0<\alpha<1,\ x\in\mathbb{R},\ t>0, \tag{1}$$

with the initial condition

$$u\left(x,0\right)=u_0\left(x\right),\ x\in\mathbb{R}, \tag{2}$$

where $u_0(x)\in L^p(\mathbb{R})$, $p\geq 1$, $f(x,t)$ is a continuously differentiable function and $f(x,0)=0$ and $D^{1-\alpha}$ represents the following Riemann-Liouville fractional derivative of order $1-\alpha$

$$D_t^{\alpha}u\left(x,t\right)=\frac{1}{\Gamma\left(1-\alpha\right)}\frac{\partial}{\partial t}\int\limits_0^t\left(t-s\right)^{-\alpha}f\left(s\right)ds.$$

In [1,2] generalized the classical Duhamel principle for the Cauchy problem to general inhomogeneous fractional distributed differential-operator equations.

**Keywords:** Duhamel Principle, diffusion equation, fractional derivative, Green's function.

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## References

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