Compactness results for the resolvent of a singular second order hyperbolic operator with a mixed derivative

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Abstract: The resolvent compactness question for differential operators defined in an unbounded domain was studied, mainly, in the one-dimensional case, or in the case of an elliptic operator. There are works [1] and [2] devoted to the discreteness of the spectrum and other important properties of the resolvent of hyperbolic and mixed type operators. However, in these papers were considered Â operators given in the Hilbert and Lebesgue spaces. At the same time, a number of practical problems lead to singular differential operators in the class of continuous functions.

Let $\overline{\Omega} = [0;\omega] \times (-\infty;\infty)$. We denote by $C_*(\overline{\Omega}, R^n)$ the space with norm

$$\|U\| = \sup_{(x,t)\in\overline{\Omega}} |U(x,t)|$$

of bounded functions $U : \overline{\Omega} \to R^n$ that are continuous on $t \in R$ for any fixed $x \in [0,\omega]$ and uniformly with respect to $t \in R$ continuous on $x \in [0,\omega]$.

In this work in $C_*(\overline{\Omega}, R^n)$ we consider the following hiperbolic operator:

$$LU = \frac{\partial^2 U}{\partial t \partial x} - A(x,t)\frac{\partial U}{\partial x} - B(x,t)\frac{\partial U}{\partial t} - C(x,t)U.$$

We present conditions for the bounded invertibility of L and the compactness of the inverse operator L^{-l}. Here $A(x,t)$, $B(x,t)$, $C(x,t)$ are the continuous $(n \times n)-$ matrixes.

Keywords: singular hiperbolic operator, space of bounded functions, well-posedness, resolvent, compactness.

2010 Mathematics Subject Classification: 35L81, 47F05, 47A10

References

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