Trigonometric system and optimal approximation

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Abstract:

n-Widths were introduced to compare and classify the power of approximation of a wide range of algorithms. Optimality of the trigonometric system is a frequently discussed topic in the theory of n-widths [1,2]. We present a new phenomenon in the behavior of the trigonometric system in the the "usual" order, i.e. $\{1, \exp(\pm i\cdot), \exp(\pm 2i\cdot), \cdots\}$. Namely, the sequence of subspaces $\mathcal{T}_n$ of trigonometric polynomials of order n is optimal in the sense of order of Kolmogorov's n-widths $d_n(K * U_p, L_q)$ on convolution classes $K * U_p$ in L_q for all $1 < p, q < \infty$ in the case of "superhigh" smoothness (analytic and entire), that is in the case

$$K(x) = \sum_{k=1}^{\infty} \exp(-\mu k^{\varrho}) \cos kx, \mu > 0, \varrho \geq 1$$

[4,5] and "supersmall" smoothness, i.e. if

$$K(x) = \sum_{k=1}^{\infty} k^{-\left(\frac{1}{p}-\frac{1}{q}\right)_+} (\log k)^{-\nu} \cos kx, \nu > 0.$$

However, in the intermediate cases of "small smoothness" $(1/p - 1/q < r < 1/p)$ and "finite smoothness" [3] $(r > 1/p)$, i.e. if

$$K(x) = \sum_{k=1}^{\infty} k^{-r} \cos kx$$

the sequence of subspaces $\mathcal{T}_n$ is not optimal for all $1 < p, q < \infty$. Also, we present similar results for linear and Bernstein's n-width.

References

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