One class of inverse problems for reconstructing the process of heat conduction from nonlocal data

Makhmud Sadybekov[1], Gulnar Dildabek[1,2], Marina B. Ivanova[1,3]

[1] *Institute of Mathematics and Mathematical Modeling, Kazakhstan,*
[2] *Al-Farabi Kazakh National University, Kazakhstan*
[3] *South-Kazakhstan State Pharmaceutical Academy, Kazakhstan*
sadybekov@math.kz, dildabek.g@gmail.com, marina-iv@mail.ru

Abstract: We consider one class of inverse problems for a one-dimensional heat equation with involution and with periodic type boundary conditions with respect to a space variable.

We will consider the process which is so slow that it is described by an heat equation. Thus, this process is described by the equation

(1) $$D_t\Phi(x,t) - \Phi_{xx}(x,t) + \varepsilon\Phi_{xx}(-x,t) = f(x)$$

in the domain $\Omega = \{(x,t): \ -\pi < x < \pi, \ 0 < t < T\}$. Here $f(x)$ is the influence of an external source that does not change with time; $t = 0$ is an initial time point and $t = T$ is a final one.

In this report, several variants of the operator D_t will be considered. Problems for a classical heat equation and for an equation with fractional derivatives with respect to a time variable are considered:

$$i) \quad D_t\Phi(x,t) = \Phi_t(x,t);$$

$$ii) \quad D_t\Phi(x,t) = t^{-\beta} D_{C\ t}^{\alpha}\Phi(x,t), \quad D_{C\ t}^{\alpha}\varphi(t) = I^{1-\alpha}\left[\frac{d}{dt}\varphi(t)\right], \ 0 < \alpha < 1.$$

Where $D_{C\ t}^{\alpha}$ is a Caputo derivative and $I^{1-\alpha}$ is a Riemann-Liouville fractional integral.

As the additional information we take values of the initial and final conditions of the temperature

(2) $$\Phi(x,0) = \phi(x), \quad \Phi(x,T) = \psi(x), \quad x \in [-\pi,\pi].$$

Since the wire is closed, it is natural to assume that the temperature at the ends of the wire is the same at all times:

(3) $$\Phi(-\pi,t) = \Phi(\pi,t), \ t \in [0,T].$$

Consider the process in which the temperature at one end at every time point t is proportional to the rate of change speed of the average value of the temperature throughout the wire. Then,

(4) $$\Phi(-\pi,t) = \gamma D_t \int_{-\pi}^{\pi} \Phi(\xi,t)\, d\xi, \ t \in [0,T].$$

Here γ is a proportionality coefficient.

Thus the investigated process is reduced to the following mathematical inverse problem: *Find a right-hand side $f(x)$ of the heat equation (1), and its solution $\Phi(x,t)$ subject to the initial and final conditions (2), the boundary condition (3), and condition (4).*

This research is financially supported by a grant AP05133271 and by the target program BR05236656 from the Science Committee from the Ministry of Science and Education of the Republic of Kazakhstan.

Keywords: Inverse problem; heat equation; equation with involution; periodic boundary conditions; method of separation of variables

2010 Mathematics Subject Classification: 35K20, 35R30, 35R10

References

[1] B. Ahmad, A. Alsaedi, M. Kirane, R.G. Tapdigoglu; An inverse problem for space and time fractional evolution equations with an involution perturbation, Quaestiones Mathematicae, vol. 40, no. 2, 151—160, 2017.