On coersive solvability of a third-order differential equation with oscillating intermediate coefficient

Kordan Ospanov[1], Zhuldyz Yeskabylova[2]

L.N. Gumiliov Eurasian National University, Kazakhstan
[1] *kordan.ospanov@gmail.com,* [2] *juli_e92@mail.ru*

Abstract: In this report the sufficient conditions of well-posedness in $L_2(-\infty, +\infty)$ of the following differential equation

$$-y''' + p(x)y' + q(x)y = f(x), \tag{1}$$

with smooth, unbounded and fast oscillating coefficients are received. We give the maximal regularity estimate

$$\|y'''\|_2 + \|py'\|_2 + \|qy\|_2 \leq C\|f\|_2$$

for the solution y ($\|\cdot\|_2$ is a norm in $L_2(-\infty, +\infty)$). Here the intermediate coefficient p is not controlled by q, and $|q|$ is an unbounded function. The conditions obtained us are close to necessary.

In the case when the domain is unbounded, although the solution of the odd-order equation (1) is smooth, but it may not belong to any Sobolev space, or its belongs to the Sobolev space is not known in advance. This is one of its main differences from the Sturm-Liouville equation. So, the smoothness problems for solutions of the equation (1) are of great interest. For such equations is important to study of separability or, in other words, the maximal regularity problem.

In the case $p = 0$ the maximal regularity problem for the equation (1) and its quasilinear generalizations was investigated by M. Otelbaev, M.B. Muratbekov, K.N. Ospanov and etc. Moreover, in their works the existence of a solution was proved, and when L is linear, the uniqueness of the solution and the important spectral and approximate properties of the resolvent L^{-1} were given.

The authors were supported by the grant no. AP05131649 of the Ministry of Education and Science of Republic of Kazakhstan.

Keywords: differential equation, unbounded domain, oscillating coeffcients, well-posedness, coercive estimate

2010 Mathematics Subject Classification: 34A30, 34B40