On one generalization of the Neumann problem for the Laplace equation

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Abstract: In the work we study a certain generalization of the classical Neumann problem with fractional order of boundary operators.

Let $0 < \alpha_m < ... < \alpha_1 < \alpha \leq 1, P_m(D) = D^\alpha + \sum\limits_{j=1}^{m} a_j D^{\alpha_j}$. In the domain $\Omega = \{x \in R^n |x| < 1\}$ we consider the following problem:

$$\Delta u(x) = 0, x \in \Omega, \tag{1}$$

$$P_m(D)\, u(x) = f(x), x \in \partial\Omega. \tag{2}$$

Here $D^\beta, \beta > 0-$ operators of a fractional order in the Hadamard sense [1].

As a solution of the problem (1) - (2) we call a function $u(x) \in C^2(\Omega) \cap C(\bar{\Omega})$, for which $D^\alpha[u](x) \in C(\bar{\Omega})$, satisfying the equation (1) and the boundary condition (2) in a classical sense.

The following proposition is true.

Theorem . Let $0 < \alpha_m < ... < \alpha_1 < \alpha, \quad a_j \geq 0, \quad j = 1, 2, ..., m, \quad f(x) \in C(\partial\Omega)$. Then for solvability of the problem (1) - (2) it is necessary and sufficient the following condition:

$$\int\limits_{\partial\Omega} f(x)\, dS_x = 0.$$

Keywords: Laplace equation,fractional derivative, the Hadamard operator, Neumann problem,integral equation, solvability

2010 Mathematics Subject Classification: 35J05, 35J25, 34B10

References

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