

# On Green's function of certain boundary-value problems for the biharmonic equation

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**Abstract:** In this paper we study the method of constructing the Green's function of the Neumann and Robin problems analogues for biharmonic equation in the unit ball. Integral representations of the Green's function of these problems are obtained.

Let  $\Omega = \{x \in R^n : |x| < 1\}$  be a unit ball,  $n \geq 2$ ,  $\partial\Omega$  – unit sphere,  $a \geq 0$ . We consider in the domain  $\Omega$  the following problem:

$$(1) \quad \Delta^2 u(x) = f(x), x \in \Omega,$$

$$(2) \quad \left( \frac{\partial}{\partial \nu} + a \right) u(x) \Big|_{\partial\Omega} = 0, \left( \frac{\partial}{\partial \nu} + a \right)^2 u(x) \Big|_{\partial\Omega} = 0,$$

where  $\nu$  is an outer normal vector to the sphere  $\partial\Omega$ . We note that the problem (1) - (2) in the case  $a = 0$  is analogue of the Neumann problem for the biharmonic equation, and it was studied in [1]. It is proved that for solvability of this problem the following condition is necessary and sufficient:

$$\int_{\partial\Omega} (1 - |x|^2) f(x) dx = 0.$$

In this paper we obtain an integral representation for the Green's function of the problem (1) - (2). In the construction of the Green's function, the explicit form of the Green's function of the Dirichlet problem for equation (1) is essentially used.

**Keywords:** biharmonic equation, Green's function, Neumann problem, Robin problem, integral representation

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## REFERENCES

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