Investigation of the solvability of initial-boundary value problems for a viscoelastic model with memory

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This is join work with Prof. V.P. Orlov.

Abstract: Let $Q_T = [0,T]\times\Omega$, where $T>0$, $\Omega\subset\mathbb{R}^n$, $n=2,3$, is a bounded domain with boundary $\partial\Omega\subset C^2$. We consider in Q_T the problem

$$(1)\quad \frac{\partial v}{\partial t}+\sum_{i=1}^{n} v_i\frac{\partial v}{\partial x_i}-\mu_0\Delta v-\mu_1 \mathrm{Div}\int_0^t e^{\frac{s-t}{\lambda}}\,\mathcal{E}(v)\,(s,z(s;t,x))\,ds+\nabla p=f;$$

$$(2)\qquad z(\tau;t,x)=x+\int_t^\tau v\,(s,z(s;t,x))\;ds,\quad \mathrm{div}\,v(t,x)=0;$$

$$(3)\qquad v(0,x)=v_0(x),\ x\in\Omega;\quad v(t,x)=0,\quad (t,x)\in[0,T]\times\partial\Omega.$$

Theorem 1 [1]. *Let $f=f_1+f_2$, where $f_1\in L_1(0,T;H)$, $f_2\in L_2(0,T;V^{-1})$ and $v_0\in H$. Then there exists a weak solution of problem* (**??**)-(1).

Let $Q=(-\infty,T]\times\Omega$, where $T>0$, $\Omega\subset\mathbb{R}^n$, $n=2,3$, is a bounded domain with boundary $\partial\Omega\subset C^2$. We consider in Q the problem

$$(4)\quad \frac{\partial v}{\partial t}+\sum_{i=1}^{n} v_i\frac{\partial v}{\partial x_i}-\mu_0\Delta v-\mu_1 \mathrm{Div}\int_{-\infty}^t e^{\frac{s-t}{\lambda}}\,\mathcal{E}(v)(s,z(s;t,x))ds\;+\nabla p=f;$$

$$(5)\qquad \mathrm{div}\,v(t,x)=0,\quad (t,x)\in Q;\quad v(t,x)=0,\quad (t,x)\in(-\infty,T]\times\partial\Omega;$$

$$(6)\qquad z(\tau;t,x)=x+\int_t^\tau v(s,z(s;t,x))\,ds,\quad t,\tau\in(-\infty,T],x\in\overline{\Omega}.$$

Theorem 2 [2]. *Let $f\in L_2(-\infty,T;V^{-1})$. Then problem (**??**)-(4) has at least one weak solution.*

Keywords: Weak solution, viscoelasticity, model with memory

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