Projector approach for constructing the zero order asymptotic solution for the singularly perturbed linear-quadratic control problem in a critical case

Galina Kurina[1], Nguyen Thi Hoai[2]

[1] *Voronezh State University, Russia,*

Institute of Law and Economics, Russia, and

Federal Research Center "Computer Science and Control" of Russian Academy of Sciences, Russia

kurina@math.vsu.ru

[2] *VNU, Hanoi University of Science, Vietnam*

nguyenthihoai@hus.edu.vn

Abstract: This report is devoted to linear-quadratic optimal control problems with a weak control, fixed left point and the state equation of the form

$$\varepsilon\frac{dx}{dt} = A(t,\varepsilon)x + \varepsilon B(t,\varepsilon)u + \varepsilon f(t,\varepsilon),\ t \in [0,T], \tag{1}$$

where $T > 0$ is fixed; $\varepsilon \geq 0$ is a small parameter; $x = x(t,\varepsilon)$ and $u = u(t,\varepsilon)$ are $m-$ dimensional and $r-$ dimensional vector functions, respectively; real-valued matrices of suitable size $A(t,\varepsilon)$, $B(t,\varepsilon)$, and $f(t,\varepsilon)$ are sufficiently smooth with respect to both parameters t and ε; moreover, the matrix $A(t,0)$ is singular.

Using the direct scheme method consisting in immediate substituting a postulated asymptotic solution into the problem condition and receiving problems for finding asymptotic terms, the zero order asymptotic solution is constructed under some conditions.

In contrast to paper [1], where the problem considered has been studied, the projector approach is applied here. This approach allows us to make the algorithm of constructing the asymptotic solution more clear and to correct some inaccuracies in [1].

Keywords: linear-quadratic optimal control problems, singular perturbations, weak controls, critical case, asymptotic expansions, boundary-layer functions, direct scheme method

2010 Mathematics Subject Classification: 49J15, 41A60, 34E15

References

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