A design of an approximate regulator for a weakly nonlinear terminal control problem using the penalty functions method

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Abstract: This report is devoted to the application of the penalty functions method to the construction of approximated terminal regulator for the next optimal control problem:

$$J(u) = \frac{1}{2}\int\limits_{t_0}^{t_1} \left(x^T Q(x,\varepsilon)x + u^T R u\right) dt \to \min_u, \tag{1}$$

$$\dot{x} = A(x,\varepsilon)x + B(x,\varepsilon)u,\ x(t_0) = x^0,\ \ x(t_1) = 0,\ \ x \in X \subset R^n,\ \ u \in R^r,$$

where $0 < \varepsilon \leq \varepsilon_0 \ll 1, Q(x,\varepsilon) = Q_0 + \varepsilon Q_1(x) \geq 0, A(x,\varepsilon) = A_0 + \varepsilon A_1(x)$, $B(x,\varepsilon) = B_0 + \varepsilon B_1(x)$. Thus, we consider the problem for a weakly nonlinear state-dependent coefficients (SDC) system. By means of introducing the quadratic penalty function

$$\frac{1}{2\mu}x^T(t_1)Fx(t_1),\ \ 0 < \mu \ll 1,\ \ F > 0,$$

problem (1) becomes a SDC optimal control problem with a free right-hand endpoint. It can be investigated on the basis of the approach proposed in [1]. First of all, using ε and μ matching, we study the asymptotics of the auxiliary equation solution. This equation is close to the differential matrix SDC Riccati equation obtained in [1]. Then, on the basis of asymptotics, an approximate terminal regulator with extrapolation properties is constructed.

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