About one inverse problem of time fractional evolution with an involution perturbation

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Abstract: We consider an inverse problem for a one-dimensional fractional evolution equation with involution and with periodic boundary conditions with respect to a space variable.

We will consider a process which is so slow that it is described by an evolutionary equation with a fractional time derivative. Thus, this process is described by equation

$$D_*^\alpha \Phi(x,t) - \Phi_{xx}(x,t) + \varepsilon \Phi_{xx}(-x,t) = f(x) \tag{1}$$

in the domain $\Omega = \{(x,t): \ -\pi < x < \pi, \ 0 < t < T\}$. Here $f(x)$ is the influence of an external source that does not change with time; $t = 0$ is an initial time point and $t = T$ is a final one; and the derivative D_*^α defined in [1] as

$$D_*^\alpha u(x,t) = D_{0t}^\alpha \big\{ u(x,t) - u(x,0) - t u_t(x,0) \big\}, \ 1 < \alpha < 2$$

is a Caputo derivative for a regular function built on the Riemann-Liouville derivative D_{0t}^α. Such a Caputo derivative allows us to impose initial conditions in a natural way.

As the additional information we take values of two initial and one final conditions of the temperature

$$\Phi(x,0) = \phi(x), \quad \Phi_t(x,0) = \rho(x), \quad \Phi(x,T) = \psi(x), \quad x \in [-\pi, \pi]. \tag{2}$$

Since the wire is closed, it is natural to assume that the temperature at the ends of the wire is the same at all times:

$$\Phi(-\pi,t) = \Phi(\pi,t), \ t \in [0,T]. \tag{3}$$

Consider a process in which the temperature at one end at every time point t is proportional to the (fractional) rate of change speed of the average value of the temperature throughout the wire. Then,

$$\Phi(-\pi,t) = \gamma D_*^\alpha \int_{-\pi}^{\pi} \Phi(\xi,t)\, d\xi, \ t \in [0,T]. \tag{4}$$

Here γ is a proportionality coefficient.

Thus the investigated process is reduced to the following mathematical inverse problem: *Find a right-hand side $f(x)$ of the subdiffusion equation (1), and its solution $\Phi(x,t)$ subject to the initial and final conditions (2), the boundary condition (3), and condition (4).*

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