Fourier method approach in mixed problems for the heat equation with involution perturbation

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Abstract: In this paper we consider a mixed problem for the parabolic equation with involution of the form

$$u_t(x,t) = u_{xx}(x,t) - \alpha u_{xx}(-x,t) - q(x)\,u(x,t), \quad -1 < x < 1, \quad t > 0; \qquad (1)$$

$$u(0,x) = \varphi(x), \quad -1 \le x \le 1; \quad u(-1,t) = 0, \quad u(1,t) = 0, \quad t \ge 0. \qquad (2)$$

In order to solve the problem we apply the separation of variables supposing $u(x,t)$ is a product:

$$u(x,t) = X(x)\,T(t).$$

Here the function $X(x)$ gives the solution to the boundary value problem

$$-X''(x) + \alpha X''(-x) + q(x)\,X(x) = \lambda X(x), \quad -1 < x < 1, \quad |\alpha| < 1,$$

$$X(-1) = 0, \quad X(1) = 0. \qquad (3)$$

Theorem 1. If the number $\sqrt{\frac{1-\alpha}{1+\alpha}}$ is not even then the system of eigenfunctions of the spectral problem (3) forms the basis in $L_2(-1,1)$.

Theorem 2. Let the following conditions be satisfied:

1) a real continuous function $q(x)$ is non-negative;

2) the number $\sqrt{\frac{1-\alpha}{1+\alpha}}$ is not even $(-1<\alpha<1)$;

3) the initial condition $\varphi(x)$ of the mixed problem (1), (2) satisfies the conditions $\varphi(-1)=\varphi(1)=0$,

Then the solution to the mixed problem (1), (2) exists, is unique and can be represented by the series

$$u(x,t)=\sum_{k=1}^{\infty} A_k e^{-\lambda_k t} X_k(x).$$

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References

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