Spectral asymptotics for the Sturm–Liouville operator with a complex-valued rapidly increasing potential

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Abstract: Let $q = p + ir$, where p, q are real functions that are summable on each interval $(0, b)$, $b > 0$. Denote by L the operator acting in the space $L^2(0, +\infty)$ by the formula $Ly = l(y) := -y'' + qy$ on functions from $D(L) = D := \{y \in L^2(0, +\infty) : \ y, y'^2(0, +\infty), y(0) = 0\}$. If $p(x) \to +\infty$ as $x \to +\infty$, then the operator L has a discrete spectrum [1]. Denote by $\{\lambda_k\}_{k=1}^{\infty}$ the eigenvalues of L enumerated in order of nondecreasing of their modules taking into account algebraic multiplicities. Let L_0 be the operator that is obtained from L for $q = p$, and R is the operator of multiplication by the function r. Then $L = L_0 + iR$. If $r(x) = o(p(x))$, $x \to +\infty$, then the operator R is infinitesimal with respect to the operator L_0 in the sense of the forms. Hence, according to the Keldysh theorem, the spectrum of the operator L is localized near a positive real half-line. If $\limsup\limits_{x\to+\infty} \frac{r(x)}{p(x)} > 0$, then Keldysh's theorem does not work. Using the method of complex scaling, it is easy to show [2] that the spectrum of the operator L is localized near the ray $\arg \lambda = \beta$ if the function q admits an analytic continuation to an angle $U_\beta = \{-\beta/2 < \arg z < 0\}$. These conditions are satisfied, for example, by functions of the form $q(x) = a_0 x^\alpha$, where $\alpha > 0, |\arg a_0| < \pi$. In this case $\lambda_k(\alpha) \sim c_0 e^{2i\theta/(2+\alpha)} k^{2\alpha/(2+\alpha)}$, $k \to +\infty$. It can be seen from this formula that for each fixed α, the eigenvalues $\lambda_k(\alpha)$ are removed from l_0 for large k and the distance is comparable with $|\lambda_k|$. In this connection the question arises: what is the order of this distance if instead of x^α, we take a faster growing function (for example, e^x, e^{e^x}, etc.)? The report is devoted to this issue.

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