On one class of problems for restoring the density of sources of the process of fractional heat conductivity with respect to initial and final temperature

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Abstract: We consider a class of problems modeling the process of determining the temperature and density of sources in the process of fractional heat conductivity with respect to given initial and finite temperatures.

Let $\Omega = \{(x,t)\,,\ 0 < x < 1,\ 0 < t < T\}$. In Ω consider a problem of finding the right-hand side $f(x)$ of the fractional heat equation

$$(1) \qquad D_{0+}^{\alpha}\left(u\left(x,\ t\right)\ -\ u\left(x,\ 0\right)\right) - u_{xx}\left(x,\ t\right) = f\left(x\right),\ \left(x,\ t\right) \in \Omega$$

and its solutions $u\left(x,t\right)$ satisfying the initial and final conditions

$$(2) \qquad u\left(x,0\right) = \varphi\left(x\right), \quad u\left(x,T\right) = \psi\left(x\right), \quad 0 \leq x \leq 1,$$

and the boundary conditions

$$(3) \qquad \left\{ \begin{array}{l} a_1 u_x\left(0,t\right) + b_1 u_x\left(1,t\right) + a_0 u\left(0,t\right) + b_0 u\left(1,t\right) = 0, \\ c_1 u_x\left(0,t\right) + d_1 u_x\left(1,t\right) + c_0 u\left(0,t\right) + d_0 u\left(1,t\right) = 0. \end{array} \right.$$

The coefficients $a_k,\ b_k,\ c_k,\ d_k$ with $k = 0, 1$ in (3) are real numbers, D_{0+}^{α} stands for the Riemann-Liouville fractional derivative of order $0 < \alpha < 1$:

$$D_{0+}^{\alpha} y\left(t\right) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{y(s)ds}{\left(t-s\right)^{\alpha}},$$

while $\varphi\left(x\right)$ and $\psi\left(x\right)$ are given functions.

Problems with general boundary conditions with respect to a spatial variable that are not strongly regular are investigated. The problem is considered independently of whether a corresponding spectral problem for an operator of multiple differentiation with not strongly regular boundary conditions has the basis property of root functions. The existence and uniqueness of classical solutions to the problem are proved.

The closest to the subject of this paper is [1], in which one case of regular but not strongly regular boundary conditions was considered.

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