The construction of a solution of a related system of the Laguerre type

Zh.N. Tasmambetov[1], R.U. Zhakhina[2]

[1] *Aktobe Regional State University named after K. Zhubanov, Kazakhstan*
tasmam@rambler.ru

[2] *Aktobe Regional State University named after K. Zhubanov, Kazakhstan*
riscul_75@mail.ru

Abstract: The systems of the Laguerre type, which were obtained from the system of Horn by means of exponential transformation, are considered. The Frobenius-Latysheva method is applied for the construction of the solution. The main theorem on the existence of four linearly independent particular solutions, which are expressed in terms of the degenerate hypergeometric function of M.R. Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ in the form of normally-regular series dependent on Laguerre polynomials of two variables, is proved.

Formulation of the problem

From the system of Horn

$$\left.\begin{aligned} x\cdot Z_{xx}+(\gamma-x)\cdot Z_x-y\cdot Z_y-n\cdot Z=0,\\ y\cdot Z_{yy}+(\gamma'-y)\cdot Z_y-x\cdot Z_x-n\cdot Z=0,\end{aligned}\right\} \tag{1}$$

by means of converting a system of Laguerre type is installed

$$\left.\begin{aligned} x^2\cdot U_{xx}-xy\cdot U_y+\left(-\frac{x^2}{4}-\frac{xy}{2}+kx+\frac{1}{4}-\alpha^2\right)\cdot U=0,\\ y^2\cdot U_{yy}-xy\cdot U_x+\left(-\frac{y^2}{4}-\frac{xy}{2}+kx+\frac{1}{4}-\beta^2\right)\cdot U=0,\end{aligned}\right\}. \tag{2}$$

where $k=(\alpha+\beta+2-2\lambda)/2$ is related with the basic Laguerre system [1]. Theorem is proved.

Theorem. The system (2) has four linearly independent partial solutions, which are expressed through the degenerate hypergeometric function of M.R. Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ in the form of normal-regular series

$$\begin{aligned} U(x,y)=\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\cdot x^{\frac{\alpha+1}{2}}\cdot y^{\frac{\beta+1}{2}}\cdot\Psi_2(-n,\alpha+1,\beta+1;x,y)=\\ =\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\cdot x^{\frac{\alpha+1}{2}}\cdot y^{\frac{\beta+1}{2}}\cdot L_{n,n}^{(\alpha,\beta)}(x,y)\end{aligned} \tag{3}$$

dependent on the Laguerre's polynomial of two variables

$$L_{n,n}^{(\alpha,\beta)}(x,y)=\Psi_2(-n,\alpha+1,\beta+1;x,y).$$

Frobenius-Latysheva method is used to proof of the theorem . Series of theorems on the necessary condition for the existence of a solution of the form are also proved (3).

Keywords: Related system, the system of the Laguerre type, the system of Horn, normal-regular solution, special curves, rank, anti rank.

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References

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