Construction of solutions of inhomogeneous systems of Jacobi type

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Abstract: The possibilities of constructing a general solution of an inhomogeneous system of second-order partial differential equations of Jacobi type, closest to ordinary differential equations of the second order are studied. The method of undetermined coefficients is used for the construction of a particular solution of the system. Specific examples established a connection between systems of Jacobi type with one partial differential equation of the second order obtained by adding two equations of the original system.

Formulation of the problem

The method of undetermined coefficients is spread for the construction of a general solution of the inhomogeneous system of Jacobi type

$$\begin{aligned} &x\cdot(1-x)\cdot Z_{xx}+[\gamma-(\alpha+1)\,x]\cdot Z_x+n(\alpha+n)\cdot Z=f_1(x,y),\\ &y\cdot(1-y)\cdot Z_{yy}+[\gamma'-(\beta+1)\,y]\cdot Z_y+m(\alpha+m)\cdot Z=f_2(x,y),\end{aligned}\tag{1}$$

where $Z=Z(x,y)$ – the general unknown, $f_l(x,y)$ $(l=1,2)$ – polynomials. Work on the distribution of this method to inhomogeneous systems of the form (1) was done M.Zh. Talipova [1].

Theorem 1. The general solution of the system (1) is represented as the sum of the general solution $\overline{Z}(x,y)$ of the corresponding homogeneous system and the particular solution $\overline{Z}_0(x,y)$ of the system (1):

$$Z(x,y)=\overline{Z}(x,y)+\overline{Z}_0(x,y).\tag{2}$$

Theorem 2. Let there be given an inhomogeneous system of Jacobi's type

$$\begin{aligned} &x\cdot(1-x)\cdot Z_{xx}+[\gamma-(\alpha+1)\,x]\cdot Z_x+n(\alpha+n)\cdot Z=\\ &=7+21x+\frac{21n(n+\alpha)}{(n-1)(n+\alpha+1)}\cdot y,\\ &y\cdot(1-y)\cdot Z_{yy}+[\gamma'-(\beta+1)y]\cdot Z_y+m(\alpha+m)\cdot Z=\\ &=7+21y+\frac{21m(m+\alpha)}{(m-1)(m+\alpha+1)}\cdot x.\end{aligned}\tag{3}$$

Then, the general solution of system (3) is represented as the sum of the general solution $\overline{Z}(x,y)$ of the of the corresponding homogeneous system and the particular solution $\overline{Z}_0(x,y)$ of the inhomogeneous system (3), that is

$$Z(x,y)=\overline{Z}(x,y)+\overline{Z}_0(x,y)=C_1Z_{10}(x)Z_{01}(y)+C_2Z_{10}(x)Z_{02}(y)+$$
$$+C_3Z_{20}(x)Z_{01}(y)+C_4Z_{20}(x)Z_{02}(y)+\left(\frac{7}{n(n+\alpha)}-\right.$$
$$\left.-\frac{21}{(n-1)(n+\alpha+1)}+\frac{21x}{(n-1)(n+\alpha+1)}+\frac{21y}{(n-1)(n+\alpha+1)}\right).$$

Keywords: system, particular solution, homogeneous, inhomogeneous, general solution, polynomial, orthogonal.

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References

[1] Talipova M.Zh. *Construction of normal solutions of inhomogeneous systems of partial differential equations of the second order*, Author's abstract. ... dis. cand. Almaty, 2007, 12 pp.