Analytical Description of Directional Crystallization with Two-Phase Regions

I.V. Alexandrova, A.A. Ivanov, D.V. Alexandrov

Ural Federal University, Russian Federation

irina.alexanderova@urfu.ru

Abstract:

The present paper is devoted to the construction of an analytical solution of a nonlinear nonstationary directional crystallization model with two moving boundaries. The nonstationary process of directional crystallization of three-component melts (solutions) is investigated in the presence of two phase transition zones located between the pure solid and liquid phases. An analytical solution of the nonlinear system of heat and mass transfer equations is found in the presence of two moving interphase boundaries. The temperature distributions, the concentrations of dissolved impurities, and the fractions of the solid and liquid phases in the two-phase zones are determined. The laws of motion of the boundaries between two-phase zones are found too. We consider the heat and mass transfer equations in the moving two-phase zone

$$\rho c \frac{\partial T_p}{\partial t} = \frac{\partial}{\partial z}\left(k(\chi)\frac{\partial T_p}{\partial z}\right) + L_V \frac{\partial \varphi_A}{\partial t}, \tag{1}$$

$$\chi \frac{\partial B}{\partial t} = D_B \frac{\partial}{\partial z}\left(\chi \frac{\partial B}{\partial z}\right) + B\frac{\partial \varphi_A}{\partial t},\ \chi \frac{\partial C}{\partial t} = D_C \frac{\partial}{\partial z}\left(\chi \frac{\partial C}{\partial z}\right) + C\frac{\partial \varphi_A}{\partial t}, \tag{2}$$

where T_p is the temperature, B and C are the concentration of impurity, $k = k_l\chi + k_s(1-\chi)$, ρ, c, k_l, k_s, D_B and D_C represent the constant coefficients. The solid phase fractions φ_A, φ_B and the liquid phase fraction χ are the time-dependent functions of the spatial coordinate z. This problem has been solved on the basis of techniques developed in our previous works [1,2].

Keywords: Moving boundary problem, partial differential equations, self-similar variables

2010 Mathematics Subject Classification: 82C26, 35R37, 80A20

References

[1] D.V. Alexandrov, Solidification with a quasiequilibrium mushy region: exact analytical solution of nonlinear model, Journal of Crystal Growth, vol. 222, 816–821, 2001.

[2] D.V. Alexandrov, I.G. Nizovtseva, A.P. Malygin, H.-N. Huang, D. Lee, Unidirectional solidification of binary melts from a cooled boundary: analytical solutions of a nonlinear diffusion-limited problem, Journal of Physics: Condensed Matter, vol. 20, 114105, 2008.