Approximation of fractional semilinear Cauchy problem in a Banach spaces

Sergey Piskarev

SRCC of Lomonosov Moscow State University Russia

piskarev@gmail.com

Abstract: This talk is devoted to approximation of fractional Cauchy problem

$$(1)\qquad \begin{aligned} &(\mathbf{D}_t^\alpha u)(t) = Au(t) + f(t, u(t)), \quad 0 < t \le T, \quad 0 < \alpha \le 1, \\ &u(0) = u^0, \end{aligned}$$

in a Banach space E, where $\mathbf{D}_t^\alpha$ is the Caputo-Dzhrbashyan derivative in time, the operator A generates exponentially bounded analytic and compact resolution family $S_\alpha(\cdot, A)$ and the function $f(\cdot, \cdot)$ is smooth in both arguments. The mild solution of (1) is defined as a function $u(\cdot) \in C([0,T]; E)$ which satisfies equation

$$(2)\qquad u(t) = S_\alpha(t, A)u^0 + \int_0^t P_\alpha(t-s, A) f(s, u(s)) ds,$$

where for any $x \in E$ one has

$$\lambda^{\alpha-1} R(\lambda^\alpha, A)x = \int_0^\infty e^{-\lambda t} S_\alpha(t) x dt, R(\lambda^\alpha, A)x = \int_0^\infty e^{-\lambda t} P_\alpha(t) x dt, Re\lambda > \omega.$$

We consider semidiscrete approximation theorem on general approximation scheme under condition of compact convergence of resolvents.

Throughout this note we mainly used techniques from our paper [1], but the right hand side is given by the function $f(\cdot, \cdot)$ defined in different way convenient for applications. So we consider approximation of operator family $P_\alpha(\cdot)$ with weak singularity.

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References

[1] Ru Liu, Miao Li and Sergey Piskarev, Approximation of Semilinear Fractional Cauchy Problem, Comput. Methods Appl. Math., vol.15, no 2, 203–212, 2015.