Fourier multipliers and embedding theorems in Sobolev-Lions type spaces and application

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Abstract

In this talk, Mikhlin and Marcinkiewicz–Lizorkin type operator-valued multiplier theorems in weighted abstract Lebesgue spaces are studied. Using these results one derives embedding theorems in E_0-valued weighted Sobolev-Lions type spaces $W_{p,\gamma}^{l}\left(\Omega;E_0,E\right)$, where E_0, E are two Banach spaces, E_0 is continuously and densely embedded into E. We prove that, there exists a smoothest interpolation space E_α, between E_0 and E, such that the differential operator D^α acts as a bounded linear operator from $W_{p,\gamma}^{l}\left(\Omega;E_0,E\right)$ to $L_{p,\gamma}\left(\Omega;E_\alpha\right)$ and the following Ehrling-Nirenberg-Gagilardo type sharp estimate holds

$$\left\|D^\alpha u\right\|_{L_{p,\boldsymbol{\gamma}}\left(\Omega;E\left(A^{1-|\alpha:l|-\mu}\right)\right)} \leq C_\mu\left[h^\mu \left\|u\right\|_{W_{p,\gamma}^{l}(\Omega;E(A),E)} + h^{-(1-\mu)}\left\|u\right\|_{L_{p,\boldsymbol{\gamma}}(\Omega;E)}\right]$$

for $u\in W_{p,\gamma}^{l}\left(\Omega;E\left(A\right),E\right)$. Finally, we consider the abstract differential equation

$$Lu=\sum_{|\alpha|=2l} a_\alpha D^\alpha u + Au + \sum_{|\alpha|<2l} A_\alpha\left(x\right)D^\alpha u + \lambda u = f, \tag{1}$$

where a_α are complex numbers, A, $A_\alpha\left(x\right)$ are linear operators in a Banach space E and λ is a complex parameter.

We show that there exists a unique solution $u\in W_{p,\gamma}^{2l}\left(\mathbb{R}^n;E\left(A\right),E\right)$ to (1) for all $f\in L_{p,\gamma}\left(\mathbb{R}^n;E\right)$ and there exists a positive constant C depend only on p and γ such that the following coercive uniform estimate holds

$$\sum_{|\alpha|\leq 2l}|\lambda|^{1-\frac{|\alpha|}{2l}}\left\|D^\alpha u\right\|_{L_{p,\gamma}(\mathbb{R}^n;E)} + \left\|Au\right\|_{L_{p,\gamma}(\mathbb{R}^n;E)} \leq C\left\|f\right\|_{L_{p,\gamma}(\mathbb{R}^n;E)}.$$

By using the separability properties of (2) we show that the corresponding Cauchy problem for the parabolic equation

$$\partial_t u + \sum_{|\alpha|=2l} a_\alpha D^\alpha u + Au = f\left(t,x\right),\ t\in\left(0,\infty\right),\ x\in\mathbb{R}^n, \tag{2}$$

$$u\left(0,x\right)=0,\ x\in\mathbb{R}^n$$

is well-posed in weighted spaces $L_{\mathbf{p},\gamma}\left(\mathbb{R}^n;E\right)$ with mixed norm, where $\mathbf{p}=\left(p,p_1\right)$.