

Completeness of roots elements of linear operators in Banach spaces and applications

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ABSTRACT

Here, the spectral properties of linear operators in Banach spaces are studied. We find sufficient conditions on structure of Banach spaces and resolvent properties that guarantee completeness of roots elements of Schatten class operators. This approach generalizes the well known result for operators in Hilbert spaces [1, Theorem 9.29]. The articles [2], [3] and [4] are devoted to this question in Banach spaces. In this paper, we disclose different sufficient condition for completeness of roots elements of linear operators. We consider the class of Banach spaces which satisfy some given conditions, but our class are wider than the class of operators considered in [2], [6] and [15]. Moreover, our method of proofs are different from proofs provided in the cited references. We find a sufficient condition on structure of Banach spaces which allow to define the trace of operators and its properties. Also, we get Carleman estimates and spectral properties of linear operators possess the quasi nuclear inverses. In application we consider nonlocal boundary value problem (BVP) for the second order differential-operator equation with top variable coefficients

$$Lu = a(x)u^{(2m)}(x) + \sum_{k=0}^{2m-1} B_k(x)u^{(k)}(x) + A(x)u(x) = f(x), \quad x \in (0, 1),$$

where a is a complex-valued function, $A(x)$, $B_k(x)$ are linear operators in a Banach space E and f is a E -valued function. In application, the boundary value problems for the abstract differential equations with variable coefficients are studied. The principal parts of the appropriate differential operators are not self-adjoint. The discreteness of spectrum and completeness of root elements of these operators are obtained.

References

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