Using the conjugate equations method for solving of inverse problems of mathematical physics and mathematical epidemiology

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Abstract: G.I.Marchuk in his work [1] dedicated to conjugate equations and their applications notes that in the XXI century the theory of conjugate equations will play an exceptional role in solving complex systems, thanks to which it will be possible to solve global problems that concern society. To study complex systems and mathematical models, a number of general approaches and principles have been developed. One of them is the principle of duality. In this principle, the concept of the conjugate operator is basic.

In general view, we write the mathematical model of the inverse problem in the operator equation form

(1) $$Au = f$$

with linear operator $A : X$ and domain $D(A)$, dense in X, i.e. $D(A) = X$. Suppose that one of the goals consideration inverse problems is to minimization of the functional of the coefficient inverse problem

(2) $$J(q, u) = \langle u, q \rangle$$

We consider the conjugate operator A^* and the conjugate equation

(3) $$A^* u^* = q$$

with element q on the right hand side defining by $J(q, u)$. Then the dual expression for the desired functional has the form

(4) $$J = \langle f, u^* \rangle$$

Thus, to calculate functional (2), it is necessary to solve the conjugate equation (3) and use formula (4). In this work, the theory of conjugate equations is used to solve the inverse problem of the continuation of potential fields in the direction of disturbing masses. This problem leads to the solution of the first kind Fredholm integral equation [2].

The next task is the inverse MTS problem [3-7]. To solve the inverse MTS problem with an additional condition for the solution, the optimal control method with the duality principle is used. The principle of duality simplifies the procedure for solving the problem. For this, a conjugate equation is introduced with the elements on the right-hand side determining the objective function of the optimal control method. Then, a gradient iterative method is constructed and a numerical solution is found.

The conjugate equations are also used to solve mathematical epidemiology problems. The paper [8] presents a methodology for the joint use of mathematical models and real data, which is an effective tool for studying complex epidemiological processes and solving practical problems based on it. A significant role here is played conjugate tasks. It is convenient to construct direct and inverse connections between models and daily real data on the distribution of the COVID - 19 coronavirus and the system organization of computing technologies using variational principles.

Keywords: Inverse problem, conjugate equation, operator equation, functional, Fredholm integral equation, mathematical geophysics, mathematical epidemiology

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