A general boundary value problem for heat and mass transfer equations with high order normal derivatives in boundary conditions

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Abstract: A Multidimensional boundary value problem of heat and mass transfer, when the boundary conditions contain higher-order derivatives was considered in [1].

In this paper, we consider the following boundary value problem:

$$\frac{\partial U_k(x,t)}{\partial t} = \lambda_k \Delta U_k(x,t),\ k = 1,2 \tag{1}$$

in the domain $Q_T \equiv \{(x', x_n, t) : x' \in R^{n-1}, x_n \in R_+, t \in]\,0, T[\}$, with the initial and boundary conditions:

$$U_k(x,t) = 0, \tag{2}$$

$$\sum_{k=1}^{2} \left[\sum_{k_n=0}^{m_l} a_{l,k_n}^{(k)} \frac{\partial^{k_n} U_k(x,t)}{\partial x_n^{k_n}} + a_k^{(l)} U_k(x,t) \right] \Bigg|_{x_n=0} = \varphi_l\left(x', t\right), \tag{3}$$

$$(x', t) \in Q_T^{(1)} = Q_T \backslash x_n, m_l \geq 1, l = 1,2,$$

where Δ is Laplace operator with respect to $x = (x_1, x_2, \ldots, x_n)$; λ_k are given positive constants, and $0 < \lambda_1 < \lambda_2$; $a_{k_n,l}^{(k)}$ $(k = 1,2)$ are given constants and $\varphi_l\left(x', t\right) \in C_{x',t}^{2,1}\left(Q_T^{(1)}\right)$.

The solution of the boundary problem (1)-(3) is found in the form of a double layer potential. Using the boundary conditions, a system of integro-differential equations (SIDE) is obtained. The characteristic part of the SIDE is solved by the method of Fourier-Laplace integral transforms. The conditions for the correctness and incorrectness of the problem, expressed in terms of the given constants and boundary conditions, are found. Using the regularization method, the SIDE is reduced to a system of Volterra-Fredholm integral equations.

Theorem. If $\varphi_l\left(x', t\right) \in C_{x',t}^{2,1}\left(Q_T^{(1)}\right)$ and $q_k = \frac{\lambda_1 z_k^2 - \lambda_2}{z_k^2 - 1} > 0$ (z_k are roots of the characteristic equation), then the solution of the problem (1)-(3) exists and $U_k(x,t) \in C_{x,t}^{m_k, \left[\frac{m_k}{2}\right]}(Q_T)$.

Keywords: heat and mass transfer, a boundary value problem, high order normal derivatives, solvability conditions, regularization

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References

[1] Ye.M. Khairullin, G.A. Tulesheva and A.S. Azhibekova, *A Multidimensional boundary value problem of heat and mass transfer, when the boundary conditions contain higher-order derivatives.* International Conference ■«Actual Problems of Analysis, Differential equations and Algebra■» // The Abstract Book. 161–162, 2019.