A solvability conditions of 2-d nonlocal boudary value problem for Poisson's operator on rectangle

Dovlet M. Dovletov

Department of Mathematics, Near East University, Nicosia, TRNC, Mersin 10, Turkey
dovlet.dovletov@gmail.com

We report that new accurate conditions of solvability for the differential and difference problems [1] are established, i.e., next theorems are proved.
Theorem 1. Let $\Pi = (0 < x < 1) \times (0 < y < \pi)$ and $f \in C(\overline{\Pi})$. If

$$\sum_{r=1}^{n} \alpha_r - \sum_{s=1}^{m} \beta_s < \frac{\sinh 1}{\sinh \zeta_n} \quad for \quad \zeta_n < \eta_1, \quad or \quad \sum_{r=1}^{n} \alpha_r < \frac{\sinh 1}{\sinh \zeta_n} \quad for \quad \zeta_n > \eta_1,$$

then, belonged under $C(\overline{\Pi}) \cap C^2(\Pi)$, classical solution u of the problem

$$\begin{cases} \Delta u(x,y) = f(x,y),\ (x,y) \in \Pi;\ \ u(x,0) = u(x,\pi) = 0,\ 0 \le x < 1, \\ u(0,y) = 0,\ \ u(1,y) = \sum\limits_{r=1}^{n} \alpha_r u(\zeta_r, y) - \sum\limits_{s=1}^{m} \beta_s u(\eta_s, y),\ 0 \le y \le \pi, \\ 0 < \zeta_1 < ... < \zeta_n < 1, \quad 0 < \eta_1 < ... < \eta_m < 1, \\ \zeta_r \ne \eta_s,\ \ \alpha_r > 0,\ \ \beta_s > 0,\ \ r = \overline{1,n},\ \ s = \overline{1,m} \end{cases} \tag{1}$$

exists, it is an a unique and a priori estimate $||u||_{W_2^2(\Pi)} \le C||f||_{L_2(\Pi)}$ holds.
Theorem 2. Let θ is less than a half of a distance between any two points $0, \zeta_1, ..., \zeta_n, \eta_1, ..., \eta_m, 1$. Let $u \in C^4(\overline{\Pi})$ is the solution of (1) when

$$\sum_{r=1}^{n} \alpha_r - \sum_{s=1}^{m} \beta_s < \left(1 + \frac{4}{\pi}\right)^{1-\zeta_n-\theta} \ if\ \zeta_n < \eta_1,\ or\ \sum_{r=1}^{n} \alpha_r < \left(1 + \frac{4}{\pi}\right)^{1-\zeta_n-\theta} \ if\ \zeta_n > \eta_1,$$

then mesh solution Y of the difference scheme (3), which is proposed in [1, p. 070021-3], approximates $u(x,y)$ by the second order of accuracy in terms of $h = \sqrt{h_1^2 + h_2^2}$ if $h_2 \to 0$ in each of the difference metrics C and W_2^2.

Keywords: 2-d nonlocal boundary value problem, Poisson's operator.
2010 Mathematics Subject Classification: 35B45, 35J05, 65N06.

References

[1] Dovlet M. Dovletov, Differential and Difference Variants of 2-d Nonlocal Boundary Value Problem with Poisson's Operator, *AIP Conf. Proc.*, **2183** (2019), 070021, DOI:10.1063/1.5136183.