Local and global dynamics of a cancer tumor growth model with multipoint structure

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Abstract

We present a phase-space analysis of a mathematical model of tumor growth with an immune responses. We consider mathematical analysis of the model equations with multipoint initial condition regarding to dissipativity, boundedness of solutions, invariance of non-negativity, local and global stability and the basins of attractions. We derive some features of behavior of the three-dimensional tumor growth models with dynamics described in terms of densities of three cells populations: tumor cells, healthy host cells and effector immune cells. We found sufficient conditions, under which trajectories from the positive domain of feasible multipoint initial conditions tend to one of equilibrium points. Here, cases of the small tumor mass equilibria-the healthy equilibrium point, the "death" equilibria have been examined. Biological implications of our results are discussed. In contrast to mentioned works, here mathematical analysis of multipoint IVP for local and global stability and the multiphase basins of attractions have been investigated. Here, we consider the following multipoint initial value problem (IVP) for dynamical system

$$\dot{T} = r_1 T\left(1 - k_1^{-1} T\right) - a_{12} NT - a_{13} TI,$$

$$\dot{N} = r_2 N\left(1 - k_2^{-1} N\right) - a_{21} NT,\ \dot{I} = \frac{r_3 IT}{k_3 + T} - a_{31} IT - d_3 I, \tag{1.1}$$

$$T(t_0) = T_0 + \sum_{k=1}^{m} \alpha_{1k} T(t_k),\ N(t_0) = N_0 + \sum_{k=1}^{m} \alpha_{2k} N(t_k), \tag{1.2}$$

$$I(t_0) = I_0 + \sum_{k=1}^{m} \alpha_{3k} I(t_k),\ t_0 \in [0,\ \eta),\ t_k \in O_\delta(t_0),$$

where $T = T(t)$, $N = N(t)$, $I = I(t)$ denote the densities of tumor cells, healthy host cells and the effector immune cells, respectively at the moment t, m is a natural number and α_{ij} are real numbers.

The assumption (1.2) is given on coefficients α_{ij} and $t_0, t_1, t_2, ...t_m$. Here, (T_0, N_0, I_0) indicate the given pre-healing vector (or pre-healing vector state) such that T_0 is small enough but N_0, I_0 are big enough. The condition (1.2) links the values of vector function $V(t) = (T(t), N(t), I(t))$ at various points $t_0, t_1, ..., t_m$ with each other by healing vector (T_0, N_0, I_0). So, we called (1.2) a multipoint IVP.