

**Michaelis-Menten dynamics of a cancer tumor growth model with  
multiphase structure**

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**Abstract**

We studied a phase-space analysis of a mathematical model of tumor growth with an immune responses. The addition of a drug term to the system can move the solution trajectory into a desirable basin of attraction. We show that the solutions of the model with a time-varying drug term approach can be evaluated by a more fruitful way in down to earth style. We studied some features of behavior of one of three-dimensional tumor growth models with dynamics described in terms of densities of three cells populations: tumor cells, healthy host cells and effector immune cells. In this paper, we examine the dynamics of one cancer growth model proposed in [1], but possessing multiphase structure, i.e. we consider the dynamical system

$$\dot{E}(t) = cT - \mu_2 E + \frac{p_1 EI}{\alpha_1 + I} + s_1 - g_1(u) E, \quad (1.1)$$

$$\dot{T}(t) = r_2(T) T - \frac{aET}{\alpha_2 + T} - g_2(u) T,$$

$$\dot{I}(t) = \frac{p_2 ET}{\alpha_3 + T} - \mu_3 I + s_2 - g_3(u) I, \quad \dot{u} + d_2 u(t) = v(t)$$

with multipoint initial condition

$$E(t_0) = E_0 + \sum_{k=1}^m \alpha_{2k} E(t_k), \quad T(t_0) = T_0 + \sum_{k=1}^m \alpha_{1k} T(t_k), \quad (1.2)$$

$$I(t_0) = I_0 + \sum_{k=1}^m \alpha_{3k} I(t_k), \quad t_0 \in [0, \delta), \quad t_k \in (0, \delta), \quad t_k > t_0,$$

where  $E = E(t)$ ,  $T = T(t)$ ,  $I = I(t)$  denote the densities of effector-cell, tumor cells and the concentration of interleukin-2 (IL-2) cells, respectively, at the moment  $t$ ,  $u(t)$  denotes the amount of drug at the tumor site at time  $t$ , this is determined by the dose given  $v(t)$ .

**Referances**

1. Kirschner D, Panetta J., Modelling immunotherapy of the tumor-immune interaction, J. Math. Biol. 1998(37), 235–52.