Poisson's operator with integral nonlocal condition

Dovlet M. Dovletov
Department of Mathematics, Near East University, Nicosia, TRNC, Mersin 10, Turkey
dovlet.dovletov@gmail.com

For $\Pi = (0,1) \times (0,\pi)$, $f \in C(\overline{\Pi})$, $\rho \in C[0,\tau_1]$, $\tau_1 \leq 1$ we got new result on

$$\begin{cases} \Delta u(x,y) = f(x,y),\ (x,y) \in \Pi;\ u(x,0) = u(x,\pi) = 0,\ 0 \leq x < 1, \\ u(0,y) = 0,\ \ u(1,y) = \int\limits_0^{\tau_1} \rho(x) u(x,y) dx,\ 0 \leq y \leq \pi. \end{cases} \tag{1}$$

Theorem 1. Let $\rho(x)$ satisfies one of the conditions:

(a) $\int\limits_0^{\tau_1} \rho(x)dx < \frac{\sinh 1}{\sinh \tau_1}$ if $\rho(x)$ does not change a sign,

(b) $\int\limits_0^{\tau_1} \rho(x)dx < \frac{\sinh 1}{\sinh \tau_0}$ if the sign changes from plus to minus in $\tau_0 \in (0,\tau_1)$,

(c) $\int\limits_0^{\tau_1} \frac{\rho(x)+|\rho(x)|}{2} dx < \frac{\sinh 1}{\sinh \tau_1}$ if behaviour of $\rho(x)$ differs from (a) and (b).

Then classical solution [1] of (1) exists and $||u||_{W_2^2(\Pi)} \leq C||f||_{L_2(\Pi)}$.

Theorem 2. Let $\rho \equiv 0$ in $[\tau_1, 1]$, $\int\limits_0^{\tau_1} \rho(x)dx < (1+\frac{4}{\pi})^{1-\tau_1-\theta}$, $\theta = \min\{\frac{\tau_1}{2}, \frac{1-\tau_1}{2}\}$, ρ satisfies (a), $i_{\tau_1} h_1 \leq \tau_1 < (i_{\tau_1}+1)h_1$ if $\tau_1 < 1$, $i_{\tau_1} + 1 = N_1$ if $\tau_1 = 1$ for

$$\begin{cases} Y_{\bar{x}x} + Y_{\bar{y}y} = f(x_i, y_j),\ (x_i,y_j) \in \Pi,\ h_1 = 1/N_1,\ h_2 = \pi/N_2, \\ Y|_{x=0} = 0,\ \ y_j \in [0,\pi],\ Y|_{y=0} = Y|_{y=\pi} = 0,\ x_i \in [0,1), \\ \sum\limits_{i=1}^{i_{\tau_1}+1} (\rho_i Y_{i,j} + \rho_{i-1} Y_{i-1,j}) h_1 - 2Y_{N_1,j} = 0, j = \overline{1, N_2-1}, \rho_i = \rho(x_i). \end{cases} \tag{2}$$

If $u \in C^4(\overline{\Pi})$ is the solution of (1) for $\rho \in C[0,1]$, then mesh solution Y approximates u by the second order of accuracy in terms of $h = \sqrt{h_1^2 + h_2^2}$ for $h_1 \leq c_0 h_2$ when $h_2 \to 0$ in each of the difference metrics C, W_2^2.

Keywords: integral nonlocal condition, Poisson's operator.
2010 Mathematics Subject Classification: 35B45, 35J05, 65N06.

References

[1] Dovlet M. Dovletov, Differential and Difference Variants of 2-d Nonlocal Boundary Value Problem with Poisson's Operator, *AIP Conf. Proc.*, **2183** (2019), 070021, DOI:10.1063/1.5136183.