New NLBVP with Poisson's operator in rectangle

Dovlet M. Dovletov
Department of Mathematics, Near East University, Nicosia, TRNC, Mersin 10, Turkey
dovlet.dovletov@gmail.com

We established a priori estimate of classical solution for new NLBVPs. Let

$$\begin{cases} \Delta u(x,y) = f(x,y),\ (x,y) \in \Pi, \\ u(x,0) = u(x,\pi) = 0,\ 0 \le x < 1, u(0,y) = 0,\ 0 \le y \le \pi, \end{cases} \tag{1}$$

$$u_x(1,y) = \sum_{r=1}^{n} \alpha_r u_x(\zeta_r, y) - \sum_{s=1}^{m} \beta_s u_x(\eta_s, y),\ 0 \le y \le \pi, \tag{2}$$

$$u_x(1,y) = \int_0^{\tau_1} \rho(x) u_x(x,y) dx,\ 0 \le y \le \pi, \tag{3}$$

$\Pi = (0,1) \times (0,\pi)$, $f \in C(\overline{\Pi})$, $\alpha_r > 0,\ \beta_s > 0,\ \zeta_r \ne \eta_s\ r = \overline{1,n},\ s = \overline{1,m}$, $0 < \zeta_1 < ... < \zeta_n < 1,\ 0 < \eta_1 < ... < \eta_m < 1,\ \rho \in C[0,\tau_1],\ \tau_1 \le 1$. Let solution of (1)-(2) or (1)-(3) $u \in C^2(\Pi) \cap C(\overline{\Pi} \setminus \partial\Pi|_{x=1})$, $u_x \in C(\partial\Pi|_{x=1})$.

Theorem 1. If $\sum_{r=1}^{n} \alpha_r - \sum_{s=1}^{m} \beta_s < \frac{\cosh 1}{\cosh \zeta_n}$ for $\zeta_n < \eta_1$, $\sum_{r=1}^{n} \alpha_r < \frac{\cosh 1}{\cosh \zeta_n}$ for $\zeta_n > \eta_1$, then a solution of (1)-(2) holds a priori estimate

$$||u||_{W_2^2(\Pi)} \le C||f||_{L_2(\Pi)}. \tag{4}$$

Theorem 2. If (a) $\int\limits_0^{\tau_1} \rho(x)dx < \frac{\cosh 1}{\cosh \tau_1}$ and $\rho(x)$ does not change a sign, or (b) $\int\limits_0^{\tau_1} \rho(x)dx < \frac{\cosh 1}{\cosh \tau_0}$ and the sign is changing from plus to minus in $\tau_0 \in (0,\tau_1)$, or $\int\limits_0^{\tau_1} \frac{\rho(x)+|\rho(x)|}{2} dx < \frac{\cosh 1}{\cosh \tau_1}$ and behaviour of $\rho(x)$ differs from (a)-(b), then a solution of (1),(3) holds a priori estimate (4).

Keywords: a priori estimate, NLBVP, Poisson's operator, rectangle.
2010 Mathematics Subject Classification: 35B45, 35J05.

References

[1] Dovlet M. Dovletov, Differential and Difference Variants of 2-d Nonlocal Boundary Value Problem with Poisson's Operator, *AIP Conf. Proc.*, **2183** (2019), 070021, DOI:10.1063/1.5136183.