On the solvability of nonlinear integral equations

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Abstract: The article explores general nonlinear equations with a parameter. Sufficient conditions for the existence of a solution of nonlinear integral equations in the form of the sum of two functions for individual values of the parameter are found.

Considered the nonlinear integral equation

$$A[\varphi(x)] = \lambda \int_a^b h(x)F[t, \varphi(t)]dt + f(x), \tag{1}$$

where $\varphi(x)$ an unknown function; $\lambda \in (-\infty, \infty)$ parameter; $f(x), h(x)$ are given continuous functions, defined at a given interval $[a, b]$; $A[\varphi(x)]$ and $F[x, \varphi(x)]$ are given continuous functions, which have continuous derivatives with respect to a functional variable φ.

Investigated the solvability of the problem where it is required to find the parameter values λ, for which the equation (1) has a non-trivial solution.

The solution to equation (1) is sought in the form

$$\varphi(x) = \varphi_0(x) + \lambda u(x), \tag{2}$$

where functions $\varphi_0(x), u(x)$, and a parameter λ to be determined. Suppose that function (2) is a solution of equations (1). Then the identity holds

$$A[\varphi_0(x) + \lambda u(x)] = \lambda \int_a^b h(x)F[t, \varphi_0(x) + \lambda u(x)]dt + f(x). \tag{3}$$

Further research was carried out according to the Lagrange finite increment formula.

Keywords: nonlinear integral equation, finite increment formula, resolvent values

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