On solvability of the nonlinear optimization problem with the limitations on the control

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Abstract: In the article the solvability of the problem of optimal control of oscillatory processes described by the integro-differential equation with the Fredholm operator, with given control limitation is investigated.

Consider the problem of minimizing the functional

$$J[u] = \int_0^1 [V(T,x) - \xi(x)]^2 dx + \beta \int_0^T p[t,u(t)]dt, \beta > 0 \tag{1}$$

on the set of solutions of boundary value problem

$$V_{tt} = V_{xx} + \lambda \int_0^T K(t,\tau)V(\tau,x)d\tau + g(t,x)f[u(t)], \quad 0 < x < 1, \ 0 < t \leq T,$$

$$V(0,x) = \psi_1, \ V_t(0,x) = \psi_2, \quad 0 < x < 1, \tag{2}$$

$$V_x(t,0) = 0, V_x(t,1) = 0, \ 0 < t < T.$$

Here the control $u(t)$ is an element of a Hilbert space $H(0,T)$, i.e. $u(t) \in H(0,T)$; $f[u(t)] \in H(0,T)$ is external influence function, which is nonlinear and monotonic with respect to the functional variable $u(t)$ i.e.

$$f_u[u(t)] \neq 0, \ \forall t \in [0,T] \tag{3}$$

functions $p[t,u(t)] \in H(0,T)$, $\xi(x) \in H(0,1)$, $g(t,x) \in H(Q)$, $Q = (0,1) \times (0,T)$, $\psi_1(x) \in H_1(0,1)$, $\psi_2(x) \in H(0,1)$, $K(t,\tau) \in H(D)$, $D = (0,T) \times (0,T)$ and number

$$K_0 = \int_0^T \int_0^T K^2(t,\tau)d\tau dt$$

are considered as given, λ is a parameter, $H_1(0,1)$ is the first order Sobolev space, T is fixed time point.

In this task, the desired control $u^0(t) \in H(0,T)$ is searched among the elements of the set

$$M = \{u(t) \in H(0,T) | \ f_u[u(t)] \neq 0, \ p_u[t,u(t)] = 0\} \tag{4}$$

which minimizes the functional (1) together with the corresponding solution $V(t, x)$ of the boundary value problem (2).

It is established that the desired control is among the solutions of the nonlinear Fredholm integral equation of the first kind. Sufficient conditions for the existence of a solution of nonlinear optimization problem are found.

Keywords: Integro-differential equation, generalized solution, functional, optimality condition, nonlinear Fredholm integral equation of the first kind.

2010 Mathematics Subject Classification: Primary 49J20; Secondary 35K20

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