The Riemann -Hilbert problem for first order elliptic systems on the plane in the Hardy space

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Abstract: Let D be a finite domain bounded by a smooth Lyapunov contour $\Gamma \in C^{1,\nu}, 0 < \nu < 1$, which is oriented positively with respect to D. Let all eigenvalues of a matrix $J \in \mathbb{C}^{l\times l}$ lie on the upper half-plane. and $G(t)$ be Holder continuous $l \times l-$ matrix- value function on Γ such that $\det G(t) \neq 0$ for all $t \in \Gamma$.

We consider the Riemann – Hilbert problem

$$\frac{\partial\phi}{\partial y} - J\frac{\partial\phi}{\partial x} = 0, \quad z = x + iy \in D,$$

$$\text{Re } G(t)\phi^+(t) = f(t), \quad t \in \Gamma,$$

in the the Hardy- Smirnov space $H^p(D)$, $1 < p < \infty$, which is defined [1] analogously to the case of usual analytic function.

It is shown that this problem is Fredholmian and its index æ given by the formula

$$æ = -\frac{1}{\pi} \arg\det G(t)\big|_\Gamma + l.$$

With the help of this result the integral representation of all solutions $\phi \in H^p(D)$ of (1) is received.

Keywords: First order elliptic system, Riemann- Hilbert problem, Hardy space, index formula, integral representation

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References

[1] Soldatov A.P., *The Hardy space of solutions to first-order elliptic systems.* Doklady Mathematics. 2007, V. 76, 2, P. 660-664.